Bifurcations from the Conductive State in the Cubical Cavity Heated from Below

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ABSTRACT: *Numerical results of natural convection in a laterally insulated cubical cavity filled with air (Pr=0.71) and heated from below are compared with those obtained with linear stability theory using a complete set of spectral functions for velocity and temperature. There is agreement in the predicted critical Rayleigh numbers and velocity fields for the different convective structures that develop at the onset of convection.*

1 INTRODUCTION

The first bifurcation observed in Rayleigh-Bénard layers corresponds to the transition from the conductive state to convection. The conductive state is characterized by the no-motion of the fluid and by a linear distribution of temperature between the hot bottom plate and the cold top plate. This situation is stable until the Rayleigh number, or the dimensionless temperature difference between both plates reaches a critical value.

The conductive state is stable if the viscosity and the thermal diffusivity of the fluid damp all possible perturbations caused by unstable density gradients. The onset of motion in cavities has been studied theoretically, experimentally and numerically by several authors (Davis 1967, Heitz and Westwater 1971, Catton 1972 and Stork and Müller 1972). These previous investigations reveal that the critical Rayleigh number depends strongly on the aspect ratio of the cavity for aspect ratios lower than 2. Critical values decrease rapidly with an increase in aspect ratio for ratios smaller than one and tend asymptotically to the value of 1708 for aspect ratios of about five. On the other hand, in laterally insulated cavities all the heat is transferred from the bottom to the top plate yielding lower critical Rayleigh numbers than for cavities of the same aspect ratio with perfectly conducting sidewalls.

In the cubical cavity with lateral adiabatic walls Catton (1972) reported a critical Rayleigh number of 3446. This theoretical value agrees with the experimental work of Heitz and Westwater (1971). For the cubical cavity with perfectly conducting lateral walls Davis (1967) and Stork and Müller (1972) reported a critical value of about 7000.

This work deals with the study of the onset of convection in a laterally perfect insulated cubical cavity filled with air. This system has been selected because it has no preferred lateral direction and allows for higher multiplicity of convective structures. Natural convection in the cubical cavity heated from below at low Rayleigh numbers ($3.5\times10^3 < Ra < 10^4$) has been numerically studied by Pallares et al. (1995) and Pallares et al. (1996). Critical Rayleigh numbers for different stable convective structures reported by these authors were determined by extrapolation in the Nusselt number versus Rayleigh number plots. These critical Rayleigh numbers and the velocity fields for two convective structures at the onset of convection are compared with those obtained by linear stability analysis.

2 MODEL

The physical situation and the coordinate system is shown in figure 1.

The two horizontal plates are isothermal and the four vertical walls are considered perfectly insulated. Compressibility effects, viscous dissipation and the variation of fluid properties with temperature have been neglected, with the only exception

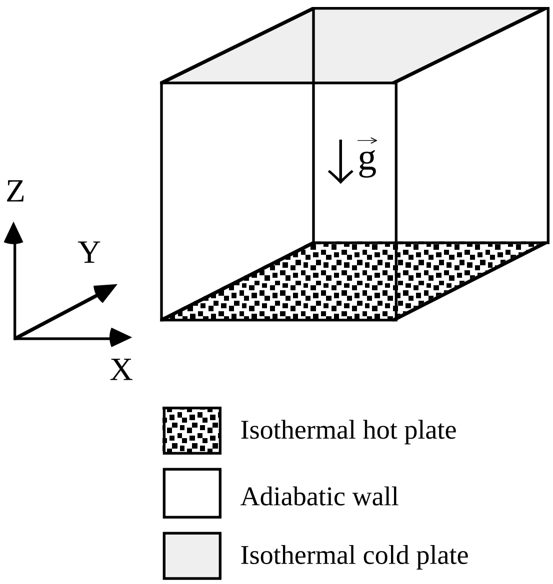

Figure 1: Physical model

of the buoyancy term, for which the Boussinesq approximation has been used.

The CFD code 3DINAMICS was used to obtain numerical solutions of the following governing equations written in dimensionless form.

$$\frac{\partial u_i}{\partial x_i} = 0 \qquad (1)$$

$$\frac{\partial u_i}{\partial t} + \frac{\partial (u_j u_i)}{\partial x_j} = -\frac{\partial P}{\partial x_i} + \Pr \frac{\partial^2 u_i}{\partial x_j x_j} + Ra \Pr T \delta_{i3} \qquad (2)$$

$$\frac{\partial T}{\partial t} + \frac{\partial (u_j T)}{\partial x_j} = \frac{\partial^2 T}{\partial x_j x_j} \qquad (3)$$

Where Ra and Pr are the Rayleigh number $(g\beta\Delta TL^3)/(\nu\alpha)$ and the Prandtl number (ν/α), respectively.

3DINAMICS uses a finite volume formulation with second order accuracy for the variation of all primitive variables with respect to time and space. A centered scheme is applied for all diffusive terms, a QUICK scheme for the convective fluxes and an ADI method for time integration. The coupling of velocities and pressure is solved following the predictor-corrector scheme SMAC and the Poisson equation for the pressure with a conjugate gradient algorithm. More details about the code and of its performance are given elsewhere (Cuesta et al. 1996).

The Galerkin method using a complete set of trial functions within the cavity was used to solve the linearized equations that govern a disturbance of the conductive state. Terms with time derivatives do not appear in the linearized equations because, in this particular problem, Sherman and Ostrach (1966) proved that instability sets in via a marginal stationary state.

The method used in selecting trial functions, is that of Catton (1972). Catton takes advantage of the fact that working with trial functions that satisfy boundary conditions and continuity eliminates pressure and reduces the linearized perturbation equations to an eigenvalue problem. However the set of trial functions used by Catton (1972) were not complete within the cavity. In this work several functions were added to those of Catton in order to achieve a complete set of trial functions. For example, the trial functions used for velocity are:

$$u = \sum_{ijk} \theta_{ijk}$$

$$\theta_{ijk} = \left\{ \begin{array}{c} \begin{pmatrix} -f_i(x)\, g_j(y)\, f'_k(z) \\ 0 \\ f'_i(x)\, g_j(y)\, f'_k(z) \end{pmatrix} \\ \text{or} \\ \begin{pmatrix} 0 \\ g_i(x)\, f_j(y)\, f'_k(z) \\ g_i(x)\, f'_j(y)\, f_k(z) \end{pmatrix} \end{array} \right\}$$

where

$$f_m(x) = \left\{ \begin{array}{c} \left(\dfrac{\cosh(\lambda_m x)}{\cosh(\lambda_m/2)} - \dfrac{\cos(\lambda_m x)}{\cos(\lambda_m/2)} \right) \\ \text{or} \\ \left(\dfrac{\sinh(\mu_m x)}{\sinh(\mu_m/2)} - \dfrac{\sin(\mu_m x)}{\sin(\mu_m/2)} \right) \end{array} \right\}$$

$$g_m(x) = \left\{ \begin{array}{c} \cos((2m-1)\pi x) \\ \text{or} \\ \sin(2m\pi x) \end{array} \right\}$$

$$\tanh(\lambda/2) + \tan(\lambda/2) = 0$$

$$\coth(\mu/2) - \cot(\mu/2) = 0$$

3 RESULTS AND DISCUSSION

A complete description and discussion of the topologies and the heat transfer rates for the steady structures obtained numerically can be found elsewhere (Pallarès et al. 1995 and Pallarès et al. 1996). Figures 2.a.1-2 and 2.b.1-2 show the dynamic and thermal fields of two (a single roll and a toroidal roll) of the five possible convective structures at Ra=10^4 and Pr=0.71 in terms of isosurfaces of the second invariant of the gradient of the velocity tensor, together with some particle paths, and in terms of isosurfaces of temperature, respectively.

The dynamic and thermal fields showed in figures a.1 and b-1 indicate that this structure is formed by a single roll motion. The other structure depicted in figures a.2 and b-2 is a toroidal roll with a single descending current at the vertical axis of the cube and four ascending currents along the vertical edges. Tis toroidal structure is an even combination of two x-rolls and two y-rolls.

Figures c-1 and c-2 show the vertical velocity contours in the horizontal mid-plane predicted numerically at the lowest Rayleigh number, while the contours obtained by linear stability analysis at the onset of convection are depicted in figures d-1 and d-2. Quantitative and qualitative agreement is found when comparing the velocitiy fields predicted by the two approximations.

Critical Rayleigh numbers for the insulated cubical cavity obtained with the complete set of spectral functions used in the present work are compared with literature values in Table I.

Table 1. Comparison of critical Rayleigh numbers

Present work. Linear stability	Catton (1972) Linear stability	Pallares et al. (1996)	Type of structure
3389	3446	3500	Single rolls
5903	-	6000	Four roll
7458	-	7800	Toroidal roll

The first critical Rayleigh number of about 3400 corresponds to the bifurcation from the conductive state to single roll convective structures similar to the one depicted in figure a-1. This critical value agrees with the one reported by Catton and with finite difference calculations. Critical Rayleigh numbers of about 6000 and 7800 have also been determined for a four roll type structure not shown in figure 2 and for the toroidal roll structure depicted in figure a-2. There is agreement between both finite difference calculations and predictions from linear stability analysis.

4 ACKNOWLEDGMENTS

The authors acknowledge the financial support received from DGICYT project no. PB93-0656-C02-01.

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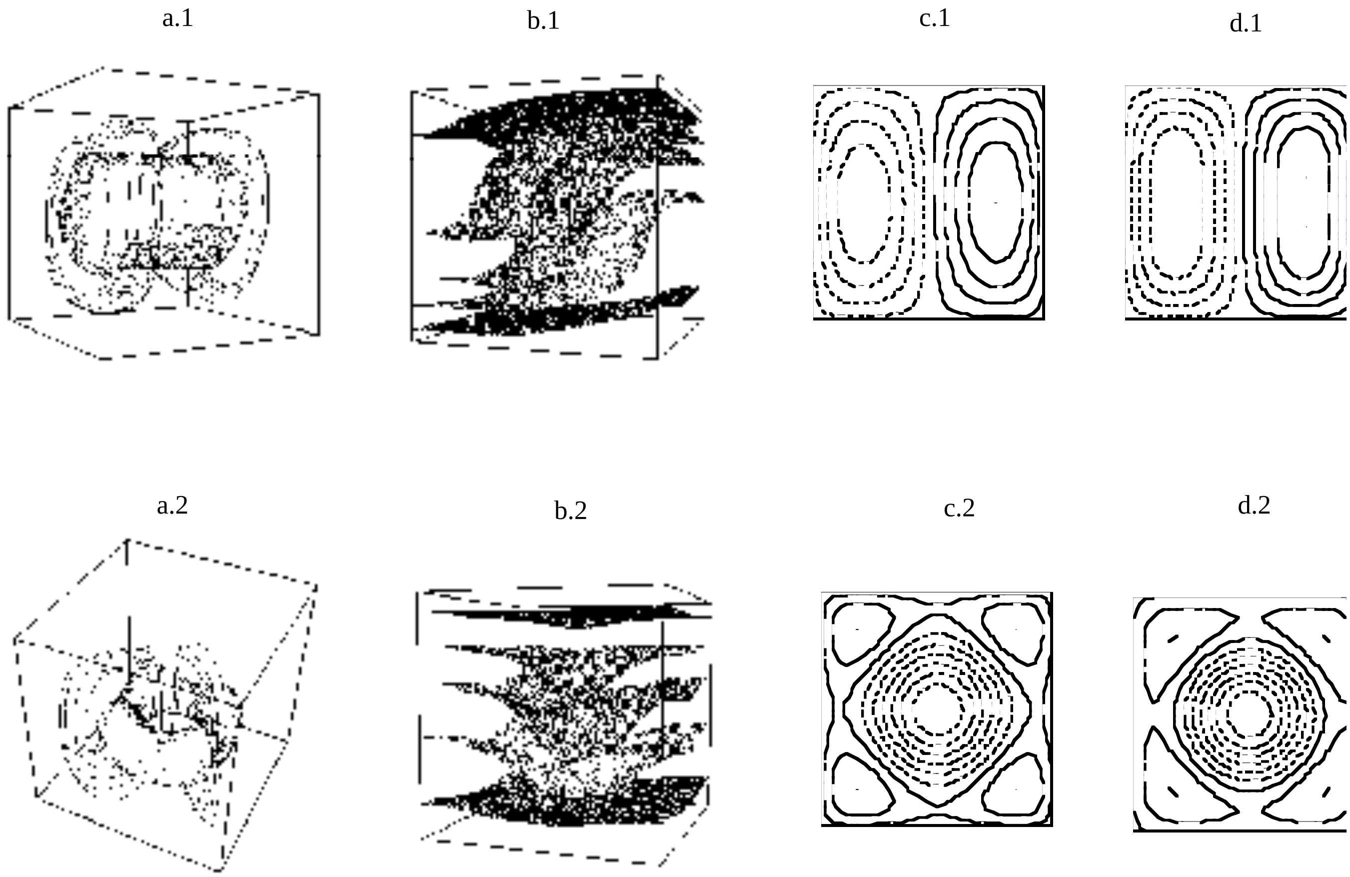

Figure 2. Two flow structures in the cubical cavity heated from below. Figures (a.1 and a.2). Isosurfaces of the second invariant of the gradient of the velocity tensor at Ra=104, a.1 P=-4000, a.2 P=-1000. Figures (b.1 and b.2). Isosurfaces of temperature at Ra=104, T=-0.4, T=-0.2, T=0, T=0.2 and T=0.4. Figures (c.1 and c.2). Numerically predicted vertical velocity contours in the horizontal midplane (z=0.5) at the onset of convection. Figures (d.1 and d.2). Contours predicted by linear stability analysis.